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# Generating a Variable from the Tail of the Normal Distribution



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# GENERATING A VARIABLE FROM THE TAIL OF THE NORMAL DISTRIBUTION

by

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Very fast procedures for generating normal variables may be based on representing the density function as a mixture, with the dominant terms chosen so that they lead to fast procedures in the computer. See [1] - [3]. There is always the problem of handling the tail of the normal distribution. Variables from the tail are needed so infrequently that convenience is a more important consideration than speed in searching for methods. The following method is very convenient - easy to understand and easy to program. At the same time, it is reasonably fast, requiring a logarithm and a square root.

The idea is to transform the tail of the normal distribution to the unit interval and then use the rejection technique. It goes as follows:

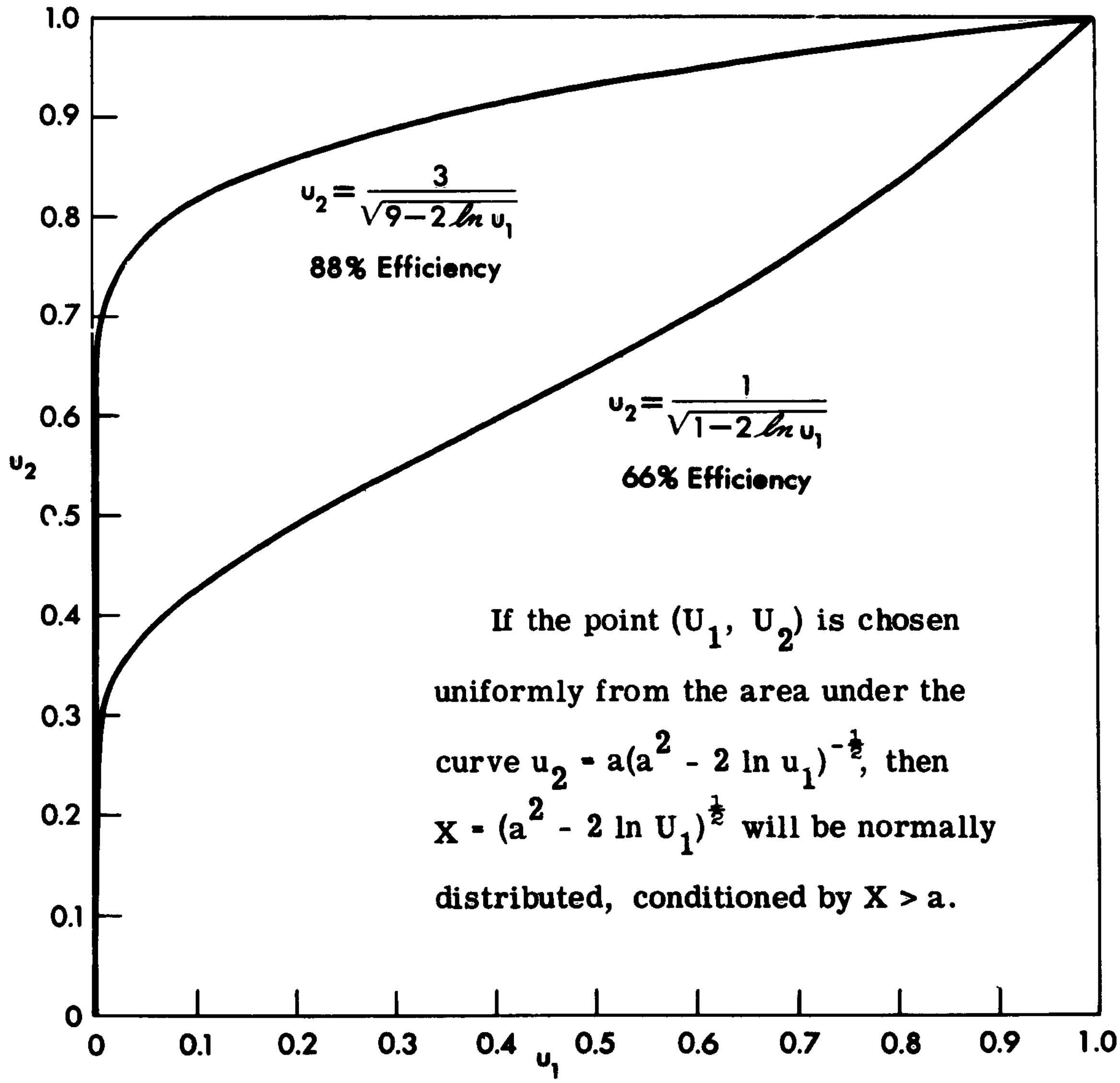
<u>To generate a standard normal variable</u> $X$, <u>conditioned by</u> $X > a$, i.e., <u>with density</u> $ce^{-.5x^2}$, $x > a$, <u>generate pairs of uniform over</u> (0,1) <u>random variables</u> $U_1, U_2$ <u>until</u>

(1) $$U_2 < a(a^2-2 \ln U_1)^{-\frac{1}{2}}$$

<u>then put</u> $X = (a^2-2 \ln U_1)^{\frac{1}{2}}$.

The density of $U_1$, given condition (1), is a multiple of $(a^2-2 \ln u_1)^{-\frac{1}{2}}$, $0 < u_1 < 1$, hence the density of $X = (a^2-2 \ln U_1)^{\frac{1}{2}}$ is a multiple of $e^{-.5x^2}$, $a < x < \infty$.

Graphs of $u_2 = a(a^2-2 \ln u_1)^{-\frac{1}{2}}$ for $a = 1$ and $3$ are plotted in this figure. When $a = 3$, the probability of event (1) is .88, so that the efficiency of the rejection technique is satisfactorily high.



## REFERENCES

[1] G. Marsaglia, "Expressing a Random Variable in Terms of Uniform Random Variables", Annals Math. Stat. 32, (1961), pp. 894-898.

[2] G. Marsaglia "Random Variables and Computers", Transactions of the Third Prague Conference, to appear.

[3] G. Marsaglia, M. D. MacLaren, T. A. Bray, "A Fast Procedure for Generating Normal Variables", Comm. of the ACM, to appear.